Jason Goldman
ORCID: 0009-0004-6743-1849
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# Collatz Structure and Period

In this work, we present a deterministic and modular structural model of the Collatz graph. Our approach defines a complete system composed of discrete composite operations, layered traversal paths, and periodic anchoring. Specifically, we establish:

- **A closed instruction set** of three composite operation types (A, B, C), based on the mod 8 residue of odd values, which fully determines traversal behavior.
- **A finite modular framework**, using mod 3 and mod 8 classifications, that governs both upward construction and downward collapse through the graph.
- **A three-dimensional coordinate system** (x, y, z), where each axis counts occurrences of a traversal type, and each odd value occupies a unique location.
- **Fully specified build and traverse mechanics**, including reversible rules and command inheritance, forming a non-overlapping, structured path lattice.
- **An internal permutation dimension** at each (x, y, z) location, encoding all valid orderings of traversal operations, constrained by system rules.
- **A recursive period and sub-period system**, originating from multiples of 3, which defines the length, coverage, and repetition of branches within the structure.

These elements together form a complete, deterministic, and verifiable framework for the behavior of all odd integers under the Collatz process.

**Standard Collatz operations:**

| | |
|---|---|
| A. 3n+1 | when n is odd |
| B. n/2 | when n is even |

These atomic operations need to be considered as a set of composite operations which operate on odd integers only. Only then can we see the structure of the system.

**New Collatz operations:**

In this form we traverse odd values only towards 1 using the following three equations:

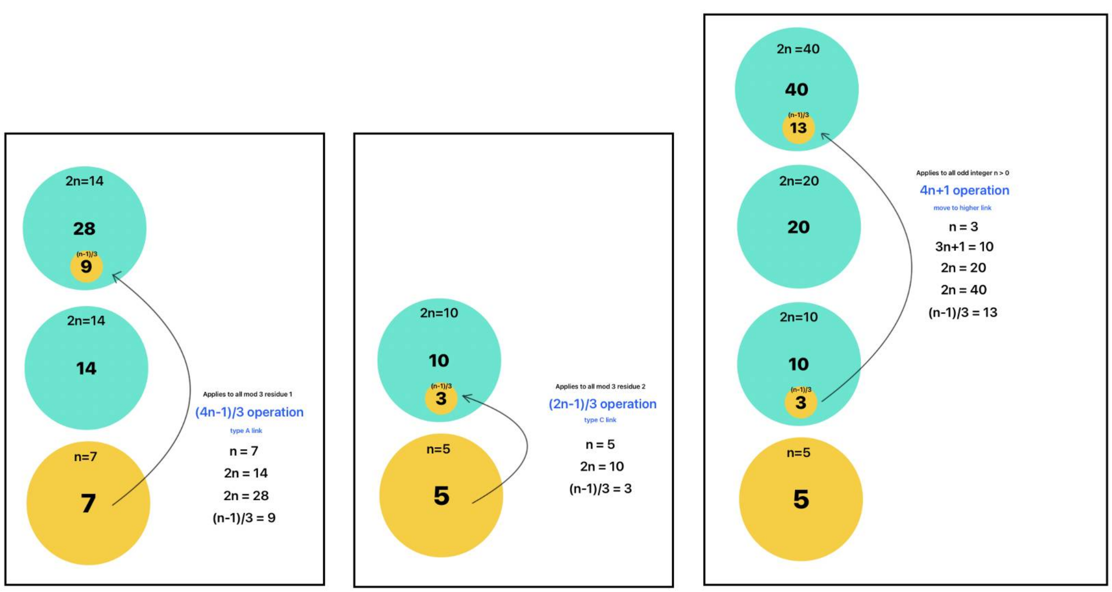
TRAVERSING TOWARDS 1

| | |
|---|---|
| A. (3n+1)/4 | when the mod 8 residue of n is 1 |
| B. (n-1)/4 | when the mod 8 residue of n is 5 |
| C. (3n+1)/2 | when the mod 8 residue of n is 3 or 7 |

These can be translated into the “build” formulas, which build the structure up from 1

BUILDING FROM 1

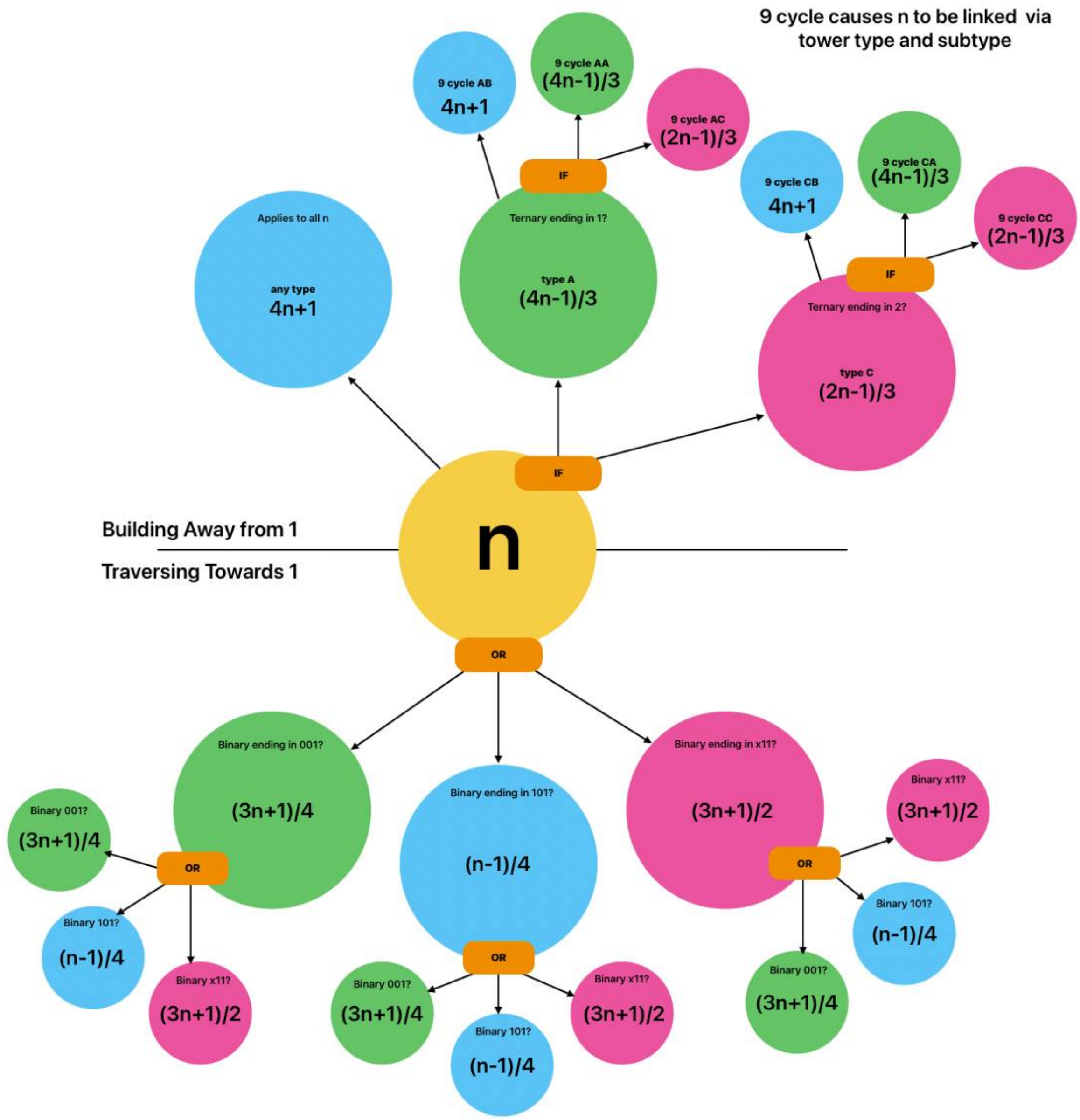
| | |
|---|---|
| A. (4n-1)/3 | when the mod 3 residue of n is 1 |
| B. 4n+1 | all n values, any mod 3 residue |
| C. (2n-1)/3 | when the mod 3 residue of n is 2 |



The possible linkages for any odd n value based upon its modular nature:

Multi step building formulas

| Type | Tail | Tail Building Formulas |
|---|---|---|
| 1 | [00]1 | ((n−1)×4^m)÷3^m+1 |
| 3,7 | [1]1 | INT((n*2^m)÷3^m) |
| 5 | 1[01] | n*2^(2×m)+(4^m−1)÷3 |



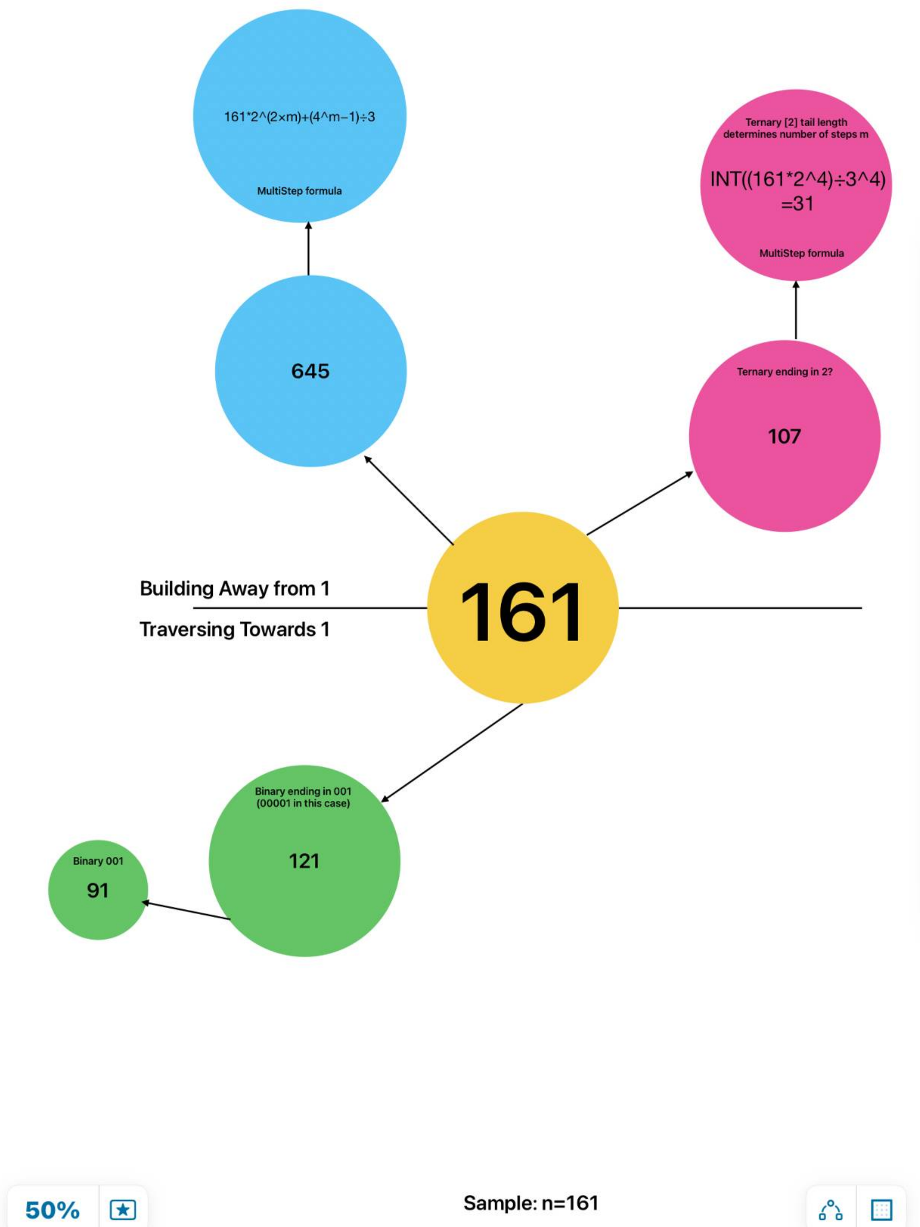

**mod 32 cycle causes n to be linked to a parent and that parents parent on traversal**

MultiStep formulas also available for traversal towards 1 based upon binary tail length of [00]1 for type 1, [01] for type 5 and 1[1] for type 3,7

Multi step traversal formulas

| Type | binary tail stripping | Tail Stripping formula |
|---|---|---|
| 1 | [00]1 | n×0.75^m−0.75^m+1 |
| 3,7 | [1]1 | n×1.5^m+1.5^m-1 |
| 5 | 1[01] | INT(n×0.25^m+0.25^m) |

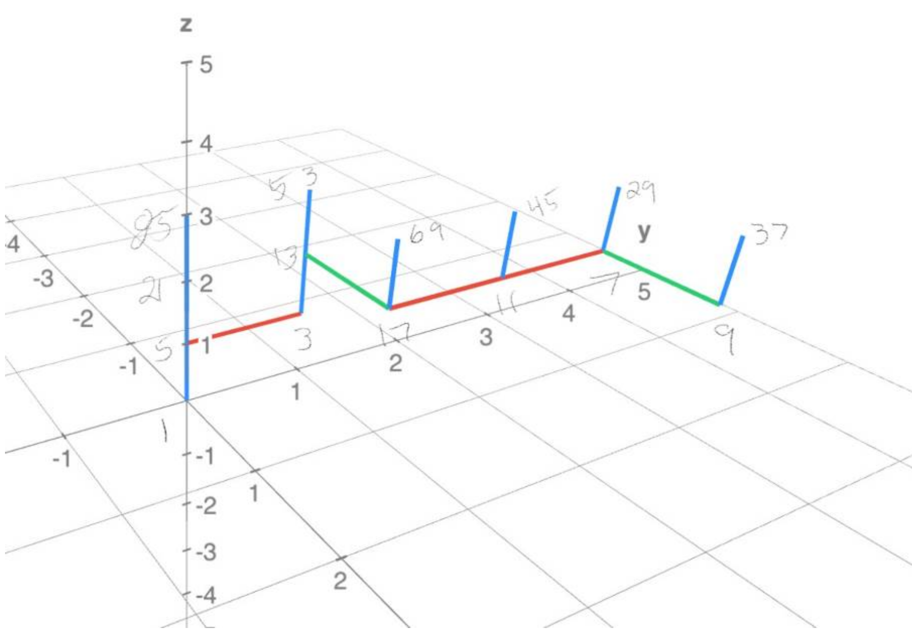
Linkages directly asserted by n=161 by its modular nature - all of these linkages can be determined by direct exam of 161 (binary/ternary nature):



**Collatz Structure:**

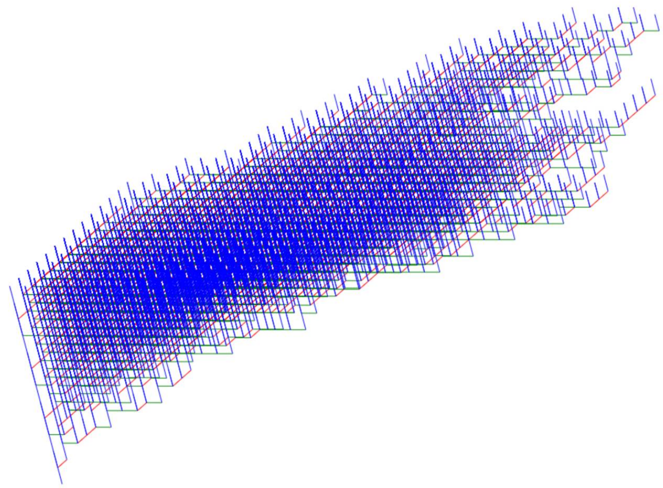
We can map any n value to an x,y,z coordinate using the following method. We will traverse its path using the new traversal formulas, and when we find mod 8 residue 1 and apply formula A we will count that as a unit movement in x. In the same manner formula C will count as a unit of movement in y and B will be a unit of z.

Seen from the build direction, starting at 1 we find the loop at 1 (type A) and a type B operation taking us to 5, which is a unit movement of z. All n values have a type B, 4n+1, z movement possible in build direction.

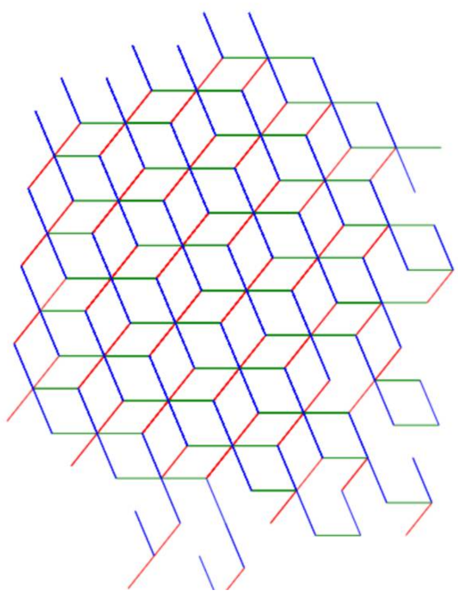



In the above image we see the path of 9 mapped, with x movement in green, y movement in red, z movement in blue. One unit of movement per formulaic step. Each point being at the x,y,z position that corresponds to the total A,C,B without consideration of the order the individual operations occurred at. We see that all values have a blue z movement springing from them, which will serve as the branch points for the layer above.

Structure with a significant number of branches built out (n=1 at bottom left):



If we isolate a two layer slice through the structure we see it forms a cubic lattice:



## Collatz Bit Planes:

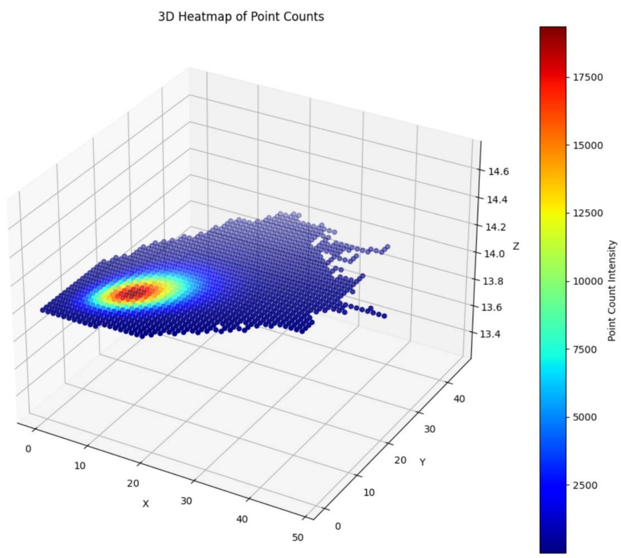
If we isolate all values of a particular bit length we find they are bound to bit planes, with a bit plane holding n values that equal in bit length to the bit plane or at most 1 bit longer (edge overlap due to the “thickness” of bit planes).

floor(2x(z-(a*x+b*y+c))) = bit plane

a = -0.2027

b = 0.2905

c = 0.5000

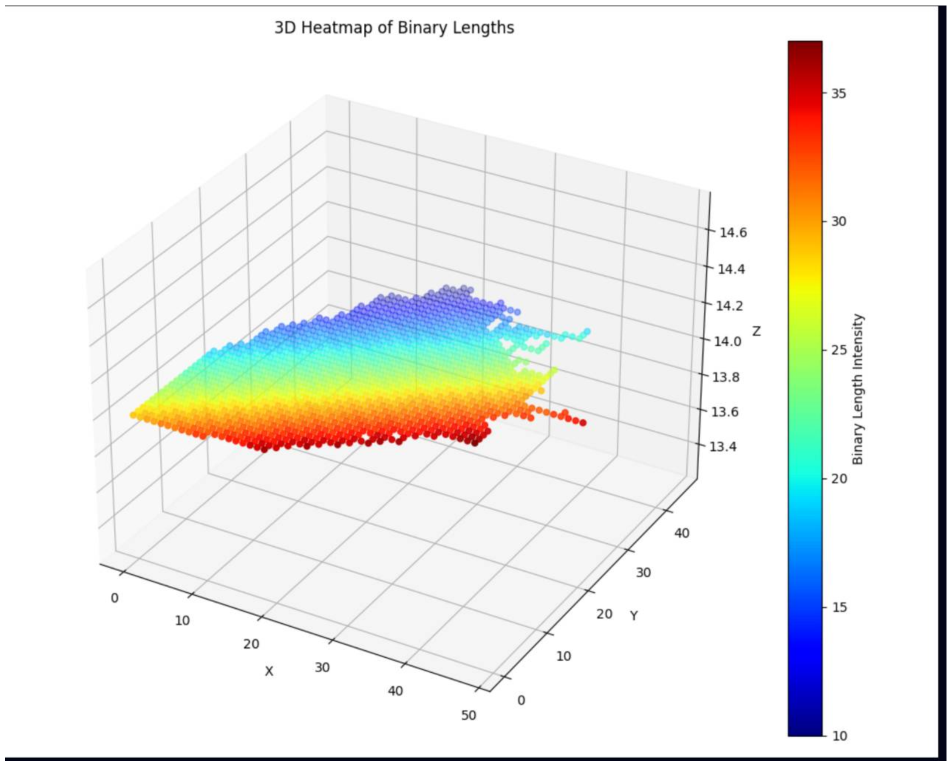



Above we see all the values of a particular bit length mapped, the shape being characteristic of all bit planes, as is the hotspot where we find many paths to the same point in x,y,z space.  This hotspot is displaying the internal dimensions of the system, as we are really graphing 3d+1 by ignoring the order of operations, with each x,y,z point holding a matrix of x,y,z orderings possible (though only certain orderings are actually possible in this system)

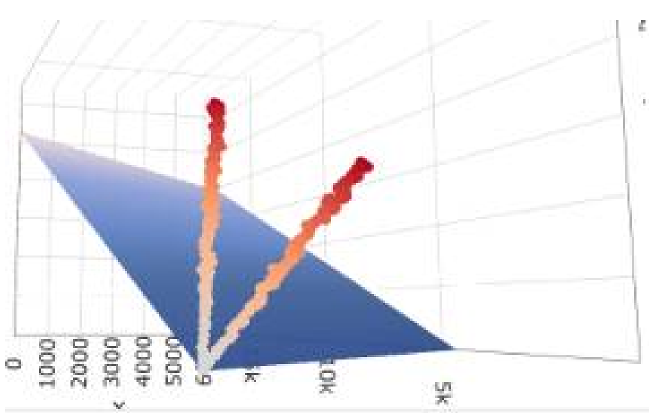
- Each (x, y, z) contains many *paths* (orderings),
- Not all permutations are valid due to system constraints,
- The **hotspot** is where **many paths converge on the same coordinate**, because multiple instruction sequences collapse into the same (x, y, z).

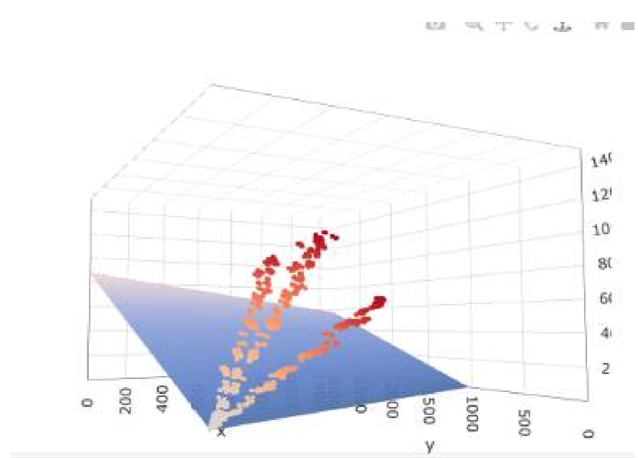
This hotspot reflects the internal permutation space that exists within each external coordinate. While we project a 3D point for each value, internally each point may represent many instruction orderings — a constrained permutation matrix.

If we view a single z layer we can see the bit planes intersect it as parallel stripes.




Here we see n values that are power of two minus one, and power of two plus one mapped, each forms a vector in this space. Similarly we see the same for power of three points, which lie above and between them. I have gone out to 5000 bits for sampling these, as well as for bit plane sampling and all holds.



**Collatz Branches:**

Before we discuss the period of the system we must reiterate the importance of branches in the system and will refer to the prior graphs for illustration.

What we see is that at the z layer 0 we have n value 1, bit length 1 at x,y,z = 0,0,0. We can use one unit of z movement (4n+1) to get to x,y,z = 0,0,1 and can continue to 0,0,infinity.

Similarly we can move from any x,y,z to x,y,infinity as all n values accept z movement.

Branches are x,y movement - traversal along a z plane. All n values from the z layer below are brought up to the next z layer as “branch bases” - all of these, having been created from 4n+1 operations are traversed using (n-1)/4 operations - they all have mod 8 residue 5 marking them.

***All branch bases are mod 8 residue 5. All mod 8 residue 5 are branch bases.***

Building a branch from its base we apply the ternary build rules - mod 3 residue driven rather than mod 8 residue as we use for traversal. We will be able to move in x and y (residue 1 and 2) and we will eventually (as we will prove in the next section) hit a residue 0, a multiple of three, where only z movement is possible, this ends the branch, it is the “branch tip”

***Mod 3 residue 0 are all branch tips. All branch tips are mod 3 residue 0.***

A branch base can be a branch tip - for example n=21 is mod 3 residue 0 and mod 8 residue 5 - it is a branch base and tip - a branch of length 1.

Branch tips, being mod 3 residue 0 are type B values. All branches have type B values as their tips, terminators of x,y movement.

Type A and C (x and y movement) make up the branch between base and tip

As all branches have mod 8 residue 5 bases and finite length to tip, containing all values, ***all values will traverse their branch to its base and drop a level of z, and repeat.*** This will always result in reaching z=0 where only the value n=1 resides.

Since all odd n must appear either as:

- A branch base (mod 8 ≡ 5),
- A node within a branch terminating at a tip (mod 3 ≡ 0),

And since the period system fully covers the B tips (3+6x), and all full combinations are accounted for with doubling and tripling structure, then **every odd value lies on some finite-length branch** which descends toward 1 through recursive application of this mechanism.

**Collatz Period and Sub-Period:**

The period of Collatz can be found by examining whole branches. This can be done by focusing in on mod 8 residue 5 values. All other values exist on branches and are controlled by the sub-period.

Examining branch length 1, which is a branch consisting only of a single type B value, we find its first appearance at 21 (the lowest n value with a path length 1)

We find that all branches at n=21+24x are also branch length 1, full branches.

The sub period is 6 (which is the space between multiple of three odds), and using the formula n=3+6x we find all n values that are type B, branch tips.

Note that the sub period identifies all n values that have same path to B (branch segment), while the period identified whole branches (base to tip) consisting of only B.

Moving on to paths of length 2, AB and CB are possible. We have doubled the number of combinations from length 1. This continues, with **each increase of 1 in path length has double the combinations of its predecessor**.

The period increases by a factor of three, as does the sub period, for each increase in branch length. Note that (Period)/(Sub Period) = 4. Ratio is always 4.

Shortest branch length 1, branch B was period 24, sub period 6.

**Period is 24*3^(branch length-1)**
**Sub period is 6*3^(branch length-1)**
**Combinations is 2^(branch length-1)**

Branch length 2, branches AB and CB are period 24*3 and sub period 6*3
- Branch AB has first occurrence at n=7 all n=7+18x values are AB path to tip.
- Branch CB has first occurrence at n=5, all n=5+18x values are CB path to tip.

The table below shows the first 6 periods, which contain their sub periods (period being "matching whole branch base to tip", sub period being "matching branch segment to tip") showing lowest n value for each combination.

Branch structure periodicity

| branch from base type 5 to tip multiple of 3 type B | 24*3^(branch length-1) | 6*3^(branch length-1) | 2^(branch length-1) | one will exist per possible combination | |
|---|---|---|---|---|---|
| **branch length** | **period of same path branches** | **sub period** | **number of combinations** | **first period n values** | **note 1** |
| 1 | 24 | 6 | 1 | 21 | 1,3,5 are first A,B,C - 3+6x will produce all B (branch tips), 1+6x will produce all A and 5+6x will produce all C - true subperiod being 3+6x here (branch tips) |
| 2 | 72 | 18 | 2 | 5, 61 | 5+18x=branches ending in CB, 7+18x=branches ending in AB |
| 3 | 216 | 54 | 4 | 85, 173, 181, 197 | |
| 4 | 648 | 162 | 8 | 29, 53, 229, 341, 373, 469, 533, 541 | 29+162=191, which is sub period, branch ending in same CAAB while 29 is period, branch consisting of CAAB |
| 5 | 1944 | 486 | 16 | 13, 37, 101, 109, 157, 269, 317, 461, 557, 773, 901, 1237, 1541, 1621, 1741, 1781 | |
| 6 | 5832 | 1458 | 32 | 125, 245, 485, 749, 757, 805, 893, 965, 1133, 1205, 1309, 1405, 1421, 1565, 1885, 2197, 2269, 2389, 2533, 2605, 3341, 3493, 3701, 3797, 4573, 4805, 4861, 4981, 4997, 5221, 5341, 5501, 5957, 6077, 6317, 6581, 6589, 6637, 6725, 6797, 6965, 7037, 7141, 7237, 7253, 7397 | |

Note that as there is a 4 to 1 ratio for period and sub period we fill find 4 lowest n sub period values per path combination, such that n*(sub period)*x where x is 0 through 3 gives all sub periods within a period.

***Each period cycle contains four sub periods, one of each residue type.***

***All PERIOD n values will be mod 8 residue 5.*** *These are all whole branches, base to tip.*

***The remaining 3 SUB PERIOD*** **values** ***will be mod 8 residue 1, 3 and 7. These represent subsets of PERIOD branches.*** *End segments from n to tip where n is not the branch base.*

***To find the lowest n values for a sub periods find the residue of the lowest n for each combination of the period using the sub period as modulus.***

For period 72, sub period 18 we have 5 as branch CB period and 61 as branch AB period. Using mod 18 we find that the first sub periods for each are 5 and 7.

period 72, sub period 18, combination 1, path to tip CB:

5+72x = all period values, base to tip whole branches

5+18x = all sub period values (5, 23, 41, 59 as first set)

period 72, sub period 18, combination 2, path to tip AB:

61+72x = all period values, base to tip whole branches

7+18x = all sub period values (7, 25, 43, 61 as first set)

Here is the first period, branch length 1. All of these n values (n=3+6x) are multiples of three and all odd multiples of three are here. They are all path to branch tip B, as they are type B (defined as n mod 3 residue 0).

In the last column we are showing sub period ands the implications of what sub period does - they account for this path to tip to come in the needed flavors - it can be one that was born of a mod 8 residue 1 value (type A), residue 3 or 7 (type C) or residue 5 (type B) - which that one out of four will be a full branch (the period), one out of four will be type A and the remaining two will be type C (both residue 3 and 7).

| 21 is the first value of period 24 and 3 is first value of sub-period 6 (21 mod sub-period). | | | | | | | |
|---|---|---|---|---|---|---|---|
| This accounts for all branch tips | | | | | | | |
| | | | | | | | sub period implies (due to n mod 8) |
| x | sub period cycle | n=3+6x | period | sub period | path to tip | n mod 8 | branch end |
| 0 | 1 | 3 | 24 | 6 | B | 3 | [C]B |
| 1 | 1 | 9 | 24 | 6 | B | 1 | [A]B |
| 2 | 1 | 15 | 24 | 6 | B | 7 | [C]B |
| 3 | 1 | 21 | 24 | 6 | B | 5 | B |
| 4 | 2 | 27 | 24 | 6 | B | 3 | [C]B |
| 5 | 2 | 33 | 24 | 6 | B | 1 | [A]B |
| 6 | 2 | 39 | 24 | 6 | B | 7 | [C]B |
| 7 | 2 | 45 | 24 | 6 | B | 5 | B |
| 8 | 3 | 51 | 24 | 6 | B | 3 | [C]B |
| 9 | 3 | 57 | 24 | 6 | B | 1 | [A]B |
| 10 | 3 | 63 | 24 | 6 | B | 7 | [C]B |
| 11 | 3 | 69 | 24 | 6 | B | 5 | B |
| 12 | 4 | 75 | 24 | 6 | B | 3 | [C]B |
| 13 | 4 | 81 | 24 | 6 | B | 1 | [A]B |
| 14 | 4 | 87 | 24 | 6 | B | 7 | [C]B |
| 15 | 4 | 93 | 24 | 6 | B | 5 | B |
| 16 | 5 | 99 | 24 | 6 | B | 3 | [C]B |
| 17 | 5 | 105 | 24 | 6 | B | 1 | [A]B |
| 18 | 5 | 111 | 24 | 6 | B | 7 | [C]B |
| 19 | 5 | 117 | 24 | 6 | B | 5 | B |

***Each sub period repeats itself on the period cycle, so that 3+24x is always the B in a CB, and 9+24x is always the B in an AB, etc.***

The first three periods and their sub periods, in detail…

| path to tip | | 24*3^(path length-1) | (period/4) | unique path | | | first n value | | Whole is mod 8 residue 5 to B |
|---|---|---|---|---|---|---|---|---|---|
| **length** | **x** | **Period** | **Sub Period** | **combinations** | **path to tip** | **y** | **lowest sub period n** | **n mod 8** | **if not whole we connect via A or C** |
| 1 | 0 | 24 | 6 | 1 | B | 0 | 3 | 3 | sub branch [C]B |
| 1 | 0 | 24 | 6 | 1 | B | 1 | 9 | 1 | sub branch [A]B |
| 1 | 0 | 24 | 6 | 1 | B | 2 | 15 | 7 | sub branch [C]B |
| 1 | 0 | 24 | 6 | 1 | B | 3 | 21 | 5 | whole branch B |
| 2 | 1 | 72 | 18 | 2 | AB | 0 | 7 | 7 | sub branch [C]AB |
| 2 | 1 | 72 | 18 | 2 | AB | 1 | 25 | 1 | sub branch [A]AB |
| 2 | 1 | 72 | 18 | 2 | AB | 2 | 43 | 3 | sub branch [C]AB |
| 2 | 1 | 72 | 18 | 2 | AB | 3 | 61 | 5 | whole branch AB |
| 2 | 1 | 72 | 18 | 2 | CB | 0 | 5 | 5 | whole branch CB |
| 2 | 1 | 72 | 18 | 2 | CB | 1 | 23 | 7 | sub branch [C]CB |
| 2 | 1 | 72 | 18 | 2 | CB | 2 | 41 | 1 | sub branch [A]CB |
| 2 | 1 | 72 | 18 | 2 | CB | 3 | 59 | 3 | sub branch [C]CB |
| 3 | 2 | 216 | 54 | 4 | AAB | 0 | 19 | 3 | sub branch [C]AAB |
| 3 | 2 | 216 | 54 | 4 | AAB | 1 | 73 | 1 | sub branch [A]AAB |
| 3 | 2 | 216 | 54 | 4 | AAB | 2 | 127 | 7 | sub branch [C]AAB |
| 3 | 2 | 216 | 54 | 4 | AAB | 3 | 181 | 5 | whole branch AAB |
| 3 | 2 | 216 | 54 | 4 | ACB | 0 | 31 | 7 | sub branch [C]ACB |
| 3 | 2 | 216 | 54 | 4 | ACB | 1 | 85 | 5 | whole branch ACB |
| 3 | 2 | 216 | 54 | 4 | ACB | 2 | 139 | 3 | sub branch [C]ACB |
| 3 | 2 | 216 | 54 | 4 | ACB | 3 | 193 | 1 | sub branch [A]ACB |
| 3 | 2 | 216 | 54 | 4 | CAB | 0 | 11 | 3 | sub branch [C]CAB |
| 3 | 2 | 216 | 54 | 4 | CAB | 1 | 65 | 1 | sub branch [A]CAB |
| 3 | 2 | 216 | 54 | 4 | CAB | 2 | 119 | 7 | sub branch [C]CAB |
| 3 | 2 | 216 | 54 | 4 | CAB | 3 | 173 | 5 | whole branch CAB |
| 3 | 2 | 216 | 54 | 4 | CCB | 0 | 35 | 3 | sub branch [C]CCB |
| 3 | 2 | 216 | 54 | 4 | CCB | 1 | 89 | 1 | sub branch [A]CCB |
| 3 | 2 | 216 | 54 | 4 | CCB | 2 | 143 | 7 | sub branch [C]CCB |
| 3 | 2 | 216 | 54 | 4 | CCB | 3 | 197 | 5 | whole branch CCB |

We can also view the period as coverage of ternary tails, which represent the encoded path to the branch tip. Ternary tail of the first period being 0 we will move on to the second - Period 72, paths AB and CB…

Period 72 Covers ternary tails 12 and 21 for whole branches (n mod 8 = 5)

| n | ternary |
|---|---|
| 5 | 12 |
| 77 | 2212 |
| 149 | 12112 |
| 221 | 22012 |
| 293 | 101212 |
| 365 | 111112 |
| 437 | 121012 |
| 509 | 200212 |
| 581 | 210112 |

| n | ternary |
|---|---|
| 61 | 2021 |
| 133 | 11221 |
| 205 | 21121 |
| 277 | 101021 |
| 349 | 110221 |
| 421 | 120121 |
| 493 | 200021 |
| 565 | 202221 |
| 637 | 212121 |

Sub Period 18 Covers All with ternary tails 12 and 21

| | decimal | | | | | ternary | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Sub Period Covers** | y=0 | y=1 | y=2 | y=3 | | y=0 | y=1 | y=2 | y=3 |
| **All with tail 12** | 5 | 23 | 41 | 59 | | 12 | 212 | 1112 | 2012 |
| **All with tail 21** | 7 | 25 | 43 | 61 | | 21 | 221 | 1121 | 2021 |

Summaries of the next two periods, 216 and 648, covering path lengths 3 and 4. These are n mod 8 residue 5 values, whole branches as we are looking at Period, which is always whole branch base (n mod 8 residue 5) to tip (n mod 3 residue 0).

Summary Period 216

| n | ternary | tail | path |
|---|---|---|---|
| 85 | 10011 | 011 | ACB |
| 173 | 20102 | 102 | CAB |
| 181 | 20201 | 201 | AAB |
| 197 | 21022 | 022 | CCB |

Summary Period 648

| n | ternary | tail | path |
|---|---|---|---|
| 29 | 1002 | 1002 | CAAB |
| 53 | 1222 | 1222 | CCCB |
| 229 | 22111 | 2111 | ACCB |
| 341 | 110122 | 0122 | CCAB |
| 373 | 111211 | 1211 | ACAB |
| 469 | 122101 | 2101 | AACB |
| 533 | 201202 | 1202 | CACB |
| 541 | 202001 | 2001 | AAAB |

The image to the right displays the periods and sub periods, with the periods outlined in black circles.

The first period and sub period in yellow, a single line of values for the single possible combination of (B) path, each 3+6y, with period occurring at 21+24x.

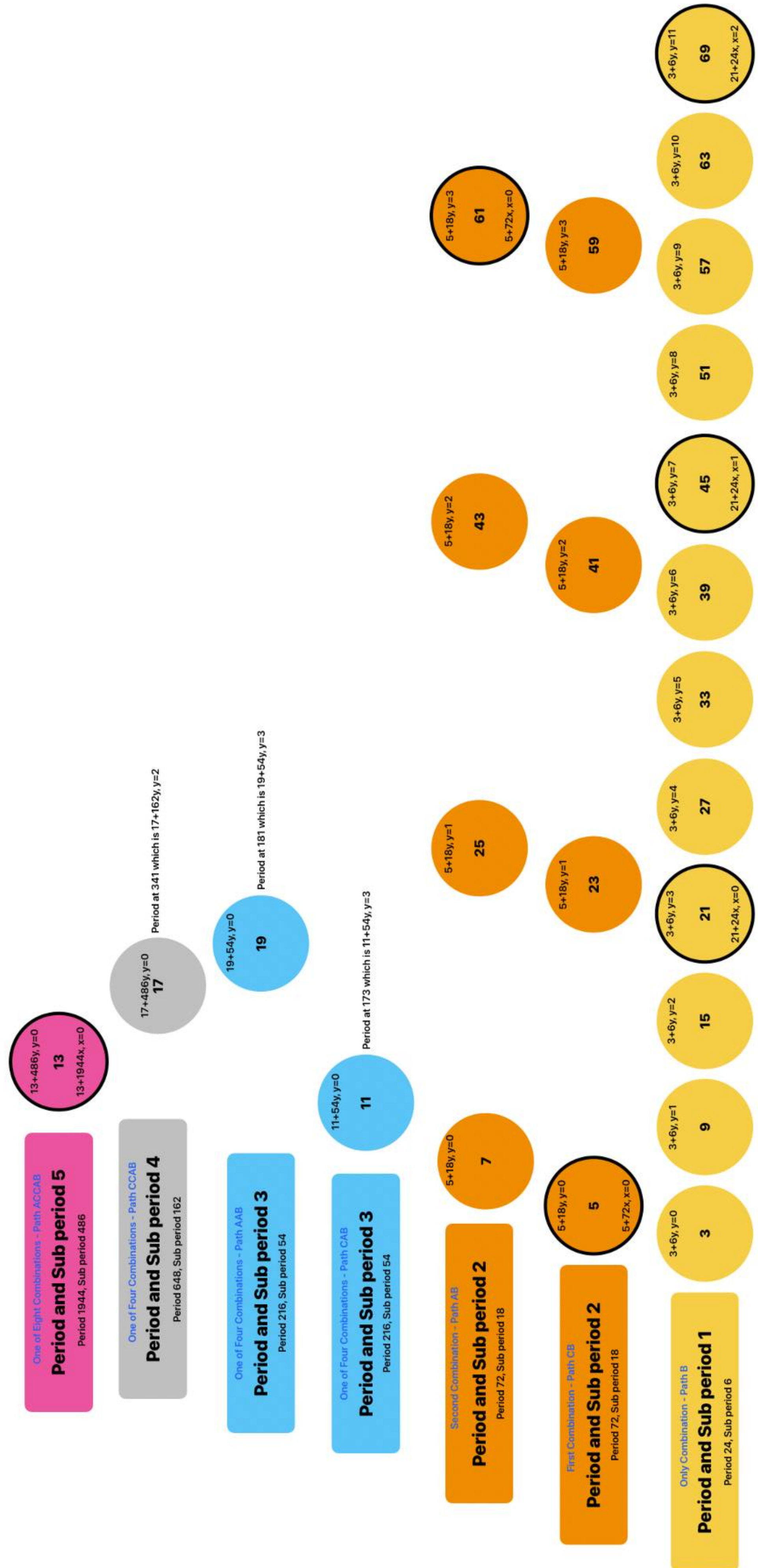
The first period describes ALL odd multiples of three, ALL branch tips.

The second in two shades of orange, one for each combination possible in a path of length 2 (AB and CB), as the second period represents.

The second period represents ALL paths ending in AB or CB, which are all paths that are not B only paths.

The third period adds another letter, allowing for combinations AAB,ACB,CAB,CCB - each having the same properties as we see here, with periods falling on whole branches and the remaining sub periods falling on partial branches - each describing ALL branches that end in that pattern.

There is no escaping this coverage, as we start with period 1 describing all branch tips and all further periods are attached to those branch tips, terminating them all.



Further, we find that when tracing a branch from tip to base the y value of the sub period always reduces with each step further from the tip.

| n | mod 8 | type | path to tip | period | sub period | y = INT((n÷sub period)) |
|---|---|---|---|---|---|---|
| 9 | 1 | B | B | 24 | 6 | 1 |
| 7 | 7 | A | AB | 72 | 18 | 0 |
| 11 | 3 | C | CAB | 216 | 54 | 0 |
| 17 | 1 | C | CCAB | 648 | 162 | 0 |
| 13 | 5 | A | ACCAB | 1944 | 486 | 0 |

| n | mod 8 | type | path to tip | period | sub period | y = INT((n÷sub period)) |
|---|---|---|---|---|---|---|
| 297 | 1 | B | B | 24 | 6 | 49 |
| 223 | 7 | A | AB | 72 | 18 | 12 |
| 335 | 7 | C | CAB | 216 | 54 | 6 |
| 503 | 7 | C | CCAB | 648 | 162 | 3 |
| 755 | 3 | C | CCCAB | 1944 | 486 | 1 |
| 1133 | 5 | C | CCCCAB | 5832 | 1458 | 0 |

| n | mod 8 | type | path to tip | period | sub period | y = INT((n÷sub period)) |
|---|---|---|---|---|---|---|
| 15 | 7 | B | B | 24 | 6 | 2 |
| 23 | 7 | C | CB | 72 | 18 | 1 |
| 35 | 3 | C | CCB | 216 | 54 | 0 |
| 53 | 5 | C | CCCB | 648 | 162 | 0 |

| n | mod 8 | type | path to tip | period | sub period | y = INT((n÷sub period)) |
|---|---|---|---|---|---|---|
| 39 | 7 | B | B | 24 | 6 | 6 |
| 59 | 3 | C | CB | 72 | 18 | 3 |
| 89 | 1 | C | CCB | 216 | 54 | 1 |
| 67 | 3 | A | ACCB | 648 | 162 | 0 |
| 101 | 5 | C | CACCB | 1944 | 486 | 0 |

| n | mod 8 | type | path to tip | period | sub period | y = INT((n÷sub period)) |
|---|---|---|---|---|---|---|
| 33 | 1 | B | B | 24 | 6 | 5 |
| 25 | 1 | A | AB | 72 | 18 | 1 |
| 19 | 3 | A | AAB | 216 | 54 | 0 |
| 29 | 5 | C | CAAB | 648 | 162 | 0 |

If we add the period 3099363912 to 445, the base of branch 27, we find we find the next period branch identical in path to 27 of course, but we also reveal more clearly the reduction in sub period y - the power of two progression showing the underlying mechanics at work.

| n | mod 8 | type | path to tip | period | sub period | y = INT((n÷sub period)) |
|---|---|---|---|---|---|---|
| 27 | 3 | B | B | 24 | 6 | 4 |
| 41 | 1 | C | CB | 72 | 18 | 2 |
| 31 | 7 | A | ACB | 216 | 54 | 0 |
| 47 | 7 | C | CACB | 648 | 162 | 0 |
| 71 | 7 | C | CCACB | 1944 | 486 | 0 |
| 107 | 3 | C | CCCACB | 5832 | 1458 | 0 |
| 161 | 1 | C | CCCCACB | 17496 | 4374 | 0 |
| 121 | 1 | A | ACCCCACB | 52488 | 13122 | 0 |
| 91 | 3 | A | AACCCCACB | 157464 | 39366 | 0 |
| 137 | 1 | C | CAACCCCACB | 472392 | 118098 | 0 |
| 103 | 7 | A | ACAACCCCACB | 1417176 | 354294 | 0 |
| 155 | 3 | C | CACAACCCCACB | 4251528 | 1062882 | 0 |
| 233 | 1 | C | CCACAACCCCACB | 12754584 | 3188646 | 0 |
| 175 | 7 | A | ACCACAACCCCACB | 38263752 | 9565938 | 0 |
| 263 | 7 | C | CACCACAACCCCACB | 114791256 | 28697814 | 0 |
| 395 | 3 | C | CCACCACAACCCCACB | 344373768 | 86093442 | 0 |
| 593 | 1 | C | CCCACCACAACCCCACB | 1033121304 | 258280326 | 0 |
| 445 | 5 | A | ACCCACCACAACCCCACB | 3099363912 | 774840978 | 0 |

| n | mod 8 | type | path to tip | period | sub period | y = INT((n÷sub period)) |
|---|---|---|---|---|---|---|
| 201326619 | 3 | B | B | 24 | 6 | 33554436 |
| 301989929 | 1 | C | CB | 72 | 18 | 16777218 |
| 226492447 | 7 | A | ACB | 216 | 54 | 4194304 |
| 339738671 | 7 | C | CACB | 648 | 162 | 2097152 |
| 509608007 | 7 | C | CCACB | 1944 | 486 | 1048576 |
| 764412011 | 3 | C | CCCACB | 5832 | 1458 | 524288 |
| 1146618017 | 1 | C | CCCCACB | 17496 | 4374 | 262144 |
| 859963513 | 1 | A | ACCCCACB | 52488 | 13122 | 65536 |
| 644972635 | 3 | A | AACCCCACB | 157464 | 39366 | 16384 |
| 967458953 | 1 | C | CAACCCCACB | 472392 | 118098 | 8192 |
| 725594215 | 7 | A | ACAACCCCACB | 1417176 | 354294 | 2048 |
| 1088391323 | 3 | C | CACAACCCCACB | 4251528 | 1062882 | 1024 |
| 1632586985 | 1 | C | CCACAACCCCACB | 12754584 | 3188646 | 512 |
| 1224440239 | 7 | A | ACCACAACCCCACB | 38263752 | 9565938 | 128 |
| 1836660359 | 7 | C | CACCACAACCCCACB | 114791256 | 28697814 | 64 |
| 2754990539 | 3 | C | CCACCACAACCCCACB | 344373768 | 86093442 | 32 |
| 4132485809 | 1 | C | CCCACCACAACCCCACB | 1033121304 | 258280326 | 16 |
| 3099364357 | 5 | A | ACCCACCACAACCCCACB | 3099363912 | 774840978 | 4 |

Doing so again we find the next instance of this branch, and note that the sub period y values have doubled.

| n | mod 8 | type | path to tip | period | sub period | y = INT((n÷sub period)) |
|---|---|---|---|---|---|---|
| 402653211 | 3 | B | B | 24 | 6 | 67108868 |
| 603979817 | 1 | C | CB | 72 | 18 | 33554434 |
| 452984863 | 7 | A | ACB | 216 | 54 | 8388608 |
| 679477295 | 7 | C | CACB | 648 | 162 | 4194304 |
| 1019215943 | 7 | C | CCACB | 1944 | 486 | 2097152 |
| 1528823915 | 3 | C | CCCACB | 5832 | 1458 | 1048576 |
| 2293235873 | 1 | C | CCCCACB | 17496 | 4374 | 524288 |
| 1719926905 | 1 | A | ACCCCACB | 52488 | 13122 | 131072 |
| 1289945179 | 3 | A | AACCCCACB | 157464 | 39366 | 32768 |
| 1934917769 | 1 | C | CAACCCCACB | 472392 | 118098 | 16384 |
| 1451188327 | 7 | A | ACAACCCCACB | 1417176 | 354294 | 4096 |
| 2176782491 | 3 | C | CACAACCCCACB | 4251528 | 1062882 | 2048 |
| 3265173737 | 1 | C | CCACAACCCCACB | 12754584 | 3188646 | 1024 |
| 2448880303 | 7 | A | ACCACAACCCCACB | 38263752 | 9565938 | 256 |
| 3673320455 | 7 | C | CACCACAACCCCACB | 114791256 | 28697814 | 128 |
| 5509980683 | 3 | C | CCACCACAACCCCACB | 344373768 | 86093442 | 64 |
| 8264971025 | 1 | C | CCCACCACAACCCCACB | 1033121304 | 258280326 | 32 |
| 6198728269 | 5 | A | ACCCACCACAACCCCACB | 3099363912 | 774840978 | 8 |

**Supporting 3D interactive visualizations:**

Interactive 3d bit plane mappings, coordinate traversal structure, and x,y,z point mapping available via links below.

Path verification tools in JS for available upon request, up to 5000-bit values have been run for bit plane sampling and vectors.  Complete runs up to 30 bits have been done for composite operation verification.

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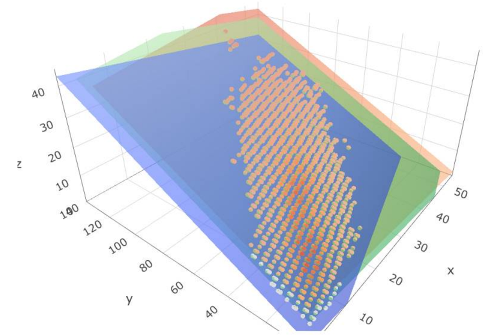
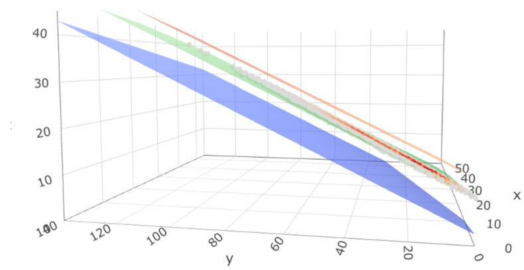
x,y,z plot of all 19 bit values:

**https://jsfiddle.net/6zhL2ejr/**

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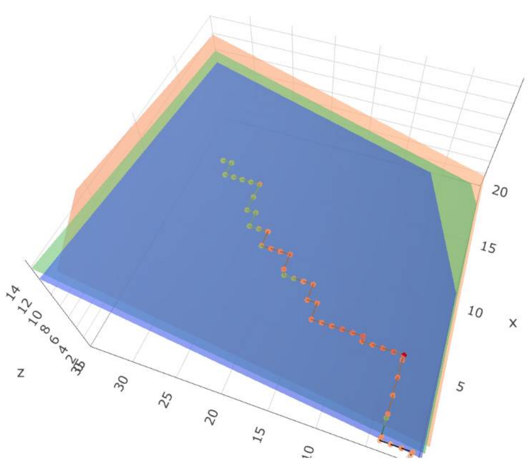
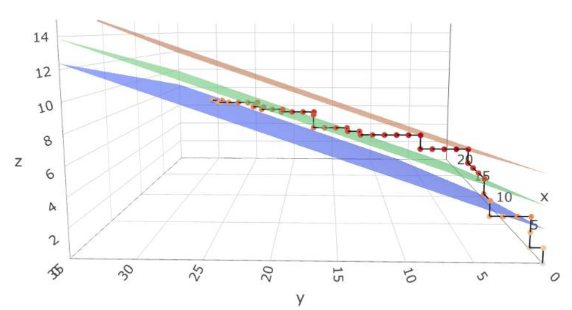
Path 27

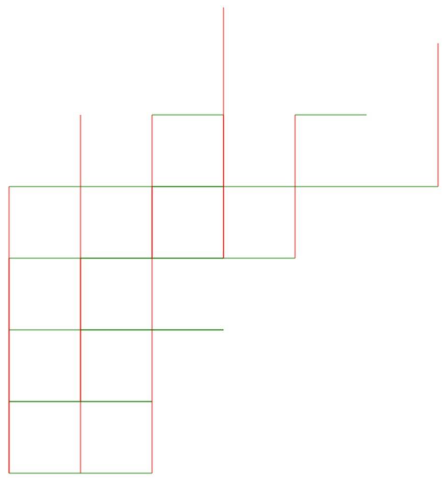
**https://jsfiddle.net/4n81tsjg/1/**

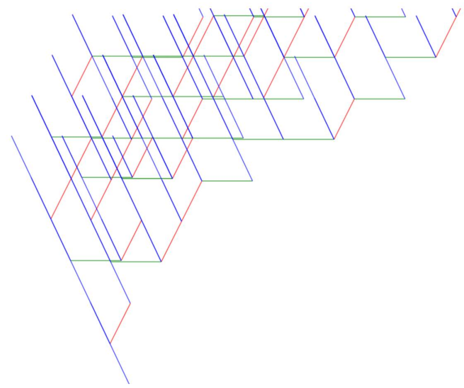


Interactive 3d structure view, builds branches up from 1 (or any seed value) - control settings via query string as explained on page.

http://gandalfpc.great-site.net/collatz/graphVectors3D.html?scale=100&branches=36&i=1

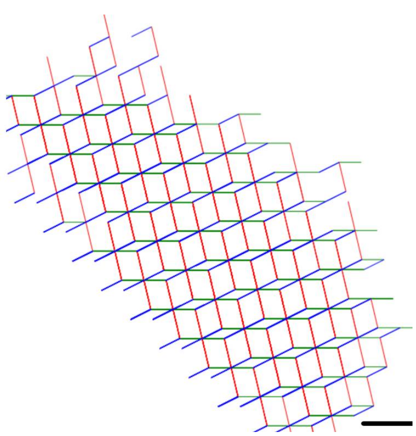


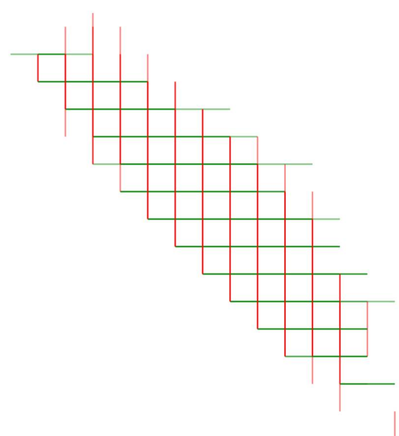


Number of branches denotes how much to build out for the structure, using the filters you can view slices of the structure such as the cubic lattice

Here we slice it by filtering with minStep=30 and maxStep=31

http://gandalfpc.great-site.net/collatz/graphVectors3D.html?branches=4000&scale=40&y=700&minStep=30&maxStep=31





# Modular-Periodic Proof of Inescapable Full Coverage in the Collatz System

## Theorem: Modular Branch Closure in Collatz System

For every odd integer $n$, there exists a finite sequence of Collatz operations that maps $n$ into a known and terminating branch of the form A/C$^k$B, where B is a multiple of 3, and the sequence structure (path) is part of a fully enumerated set derived by fixed modular periods.

## Definitions

- Let $T(n)$ be the Collatz function restricted to odd integers, using:
  - $A(n) = \frac{2n-1}{3}$ when $2n - 1 \equiv 0 \pmod 3$,
  - $C(n) = \frac{4n-1}{3}$ when $4n - 1 \equiv 0 \pmod 3$,
  - $B(n) \equiv 0 \pmod 3$: terminal nodes.
- Define a **path** as a finite string over $\{A, C\}$ ending in B.
- Let **period** $P_k$ be the smallest interval between unique base values of path length $k$.
- Define **sub-period** $S_k = P_k/4$, spacing values that follow the same path structure.

## Lemma 1: Periodic Structure Ensures All Path Combinations Exist

*Proof.* For each path length $k$, there are $2^k$ possible $A/C$ combinations. The system yields exactly $2^k$ distinct base values with unique paths of length $k$. Given a base value $n_0$ with path $p$, the set $n_0 + mS_k$ (for $m \in \mathbb{Z}$) yields new values with the same path structure. Thus, every path type exists and replicates across the number line in sub-periods of $S_k$. Therefore, the system captures all path types at every level $k$. □



## Lemma 2: All Odd Integers Lie Within Some Periodic Substructure

*Proof.* Any odd $n$ satisfies either:

- $n \equiv 5 \pmod 8$ and $n \equiv 0 \pmod 3$: belongs to the B-only sequence of period 24 (e.g., 21, 45, 69, ...),
- or falls within a mixed branch with a defined $A/C$ sequence ending in $B$.

All such values are captured by branches forming periods $P_k$ and their sub-periods $S_k$. Since $P_k$ grows exponentially and adds density to covered values, every sufficiently large odd $n$ lies within or is absorbed into this structure. □

## Lemma 3: Recursive Descent to 1 is Guaranteed

*Proof.* All branches terminate in a value divisible by 3, marked $B$. Each $B$ value arises from a finite $A/C$ sequence, implying a finite-length backtrack to a branch base. Since base values are part of a known recursive structure and lead downward (average contraction under Collatz dynamics), all paths eventually descend to 1. □

## Conclusion

By modular construction:

- Every $A/C$ path structure exists and fills via sub-periodic intervals,
- All odd integers fall into, or generate, branches in the periodic grid,
- Every such branch descends to a $B$ and ultimately to 1.

Thus, **the Collatz process covers all odd integers with inescapable completeness**, and **every path terminates**.

**Theorem (Branch Coverage):**

Let `n` be any odd integer. Then there exists a finite-length instruction sequence composed of composite operations A, B, and C, such that `n` lies on a branch ending in a type B branch tip (mod 3 ≡ 0), and can be traversed down to `n=1` via repeated application of the structure's traversal rules.